

OCU-PHYS-162

October 1 1996

Self-consistent determination of hard modes in hot QCD

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Abstract

We determine self-consistently the hard-quark and hard-gluon modes in hot QCD. The damping-rate part in resummed hard-quark or hard-gluon propagators, rather than the thermal-mass part, plays the dominant role.

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1 Introduction

It has been established by Pisarski and Braaten Since it has been realized that, within the hard-thermal-loop (HTL) resummation scheme [1, 2] of perturbative hot QCD, the damping rate for a moving *particle* diverges logarithmically, the damping rate has continuously attracted much interest [1, 3, 4, 5]. The *particle* (quark or gluon) is a “good” or stable mode in vacuum QCD. Then, the above-mentioned diverging damping rate indicates that, at nonzero temperature ($T \neq 0$), this particle “damps” instantaneously.

Landsman has pointed out [6] that the particles in vacuum theory are *not* “good” modes in thermal field theories. On the basis of a group-theoretical analysis, he then has proposed a notion of a non-shell particles as “good” or in a sense stable modes at $T \neq 0$. On the other hand, Umezawa and his coworkers have introduced [7] “thermal quasiparticles”. In both approaches, the “good” modes are designed to be determined essentially in self-consistent manners. As to the soft modes, the HTL-resummed effective propagators [1, 2] summarize the “good” modes. For hard modes, although not fully comprehensive, studies along this line have been pursued, e.g., in [4, 7, 8].

The purpose of this paper is to determine the “good” hard modes ($Q^\mu = O(T)$) to leading order at logarithmic accuracy within HTL-resummation scheme of perturbative hot QCD. By “logarithmic accuracy” we mean that the factor of $O\{1/\ln(g^{-1})\}$ is ignored when compared to the factor of $O(1)$. We work in massless SU(N) “QCD” with N_f quarks.

2 Preliminary

We start with defining the quasifree Lagrangian density for the “good” modes,

$$\mathcal{L}_0 = \mathcal{L}_0^{(q)} + \mathcal{L}_0^{(g)} + \mathcal{L}_0^{(FP)}, \quad (2.1)$$

$$\mathcal{L}_0^{(q)} = \bar{\psi} \left[i\not{\partial} - \Sigma_F(i\partial) \right] \psi, \quad (2.2)$$

$$\begin{aligned} \mathcal{L}_0^{(g)} = & -\frac{1}{4} \left(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a \right) \left(\partial^\mu A^{a\nu} - \partial^\nu A^{a\mu} \right) \\ & + \frac{1}{2} A_\mu^a \Pi_F^{\mu\nu}(i\partial) A_\nu^a - \frac{1}{2\eta} \left(\partial^\mu A_\mu^a \right) \left(\partial^\nu A_\nu^a \right), \end{aligned} \quad (2.3)$$

$$\mathcal{L}_0^{(\text{FP})} = (\partial^\mu \bar{\eta}^a) \left(\partial_\mu \eta^a - g f^{abc} A_\mu^b \eta^c \right) - \bar{\eta}^a \Pi_F^g(i\partial) \eta^a, \quad (2.4)$$

where “FP” stands for Faddeev-Popov ghost field. $\Sigma_F(i\partial)$ in (2.2) is a 4×4 matrix function of $i\partial$, which may be decomposed as

$$\Sigma_F(Q) = f(Q) \not{Q} + g(Q) \gamma^0. \quad (2.5)$$

Similarly $\Pi_F^{\mu\nu}(i\partial)$ in (2.3) may be decomposed as [9]

$$\begin{aligned} \Pi_F^{\mu\nu}(Q) &= \mathcal{P}_T^{\mu\nu}(Q) \Pi_F^T(Q) + \mathcal{P}_L^{\mu\nu}(Q) \Pi_F^L(Q) \\ &+ \mathcal{C}^{\mu\nu}(Q) \Pi_F^C(Q) + \mathcal{D}^{\mu\nu}(Q) \Pi_F^D(Q). \end{aligned} \quad (2.6)$$

Here

$$\mathcal{P}_T^{\mu\nu}(Q) \equiv - \sum_{i,j=1}^3 g_{\mu i} g_{\nu j} [\delta^{ij} - \hat{q}^i \hat{q}^j] \quad (2.7)$$

$$\mathcal{P}_L^{\mu\nu}(Q) \equiv g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2} - \mathcal{P}_T^{\mu\nu}(Q), \quad (2.8)$$

$$\mathcal{C}^{\mu\nu}(Q) \equiv \frac{1}{\sqrt{2}q_0 q} \left[Q^\mu \tilde{Q}^\nu + Q^\nu \tilde{Q}^\mu + 2q^2 \frac{Q^\mu Q^\nu}{Q^2 + i0^+} \right], \quad (2.9)$$

$$\mathcal{D}^{\mu\nu}(Q) \equiv \frac{Q^\mu Q^\nu}{Q^2 + i0^+}, \quad (2.10)$$

where $\hat{\mathbf{q}} \equiv \mathbf{q}/q$ with $q \equiv |\mathbf{q}|$ and $\tilde{Q}^\mu \equiv (0, \mathbf{q})$. $\mathcal{P}_T^{\mu\nu}(Q)$ [$\mathcal{P}_L^{\mu\nu}(Q)$] is the projection operator onto the transverse [longitudinal] mode. As is well known [9], BRS invariance of the full QCD Lagrangian leads to (see below)

$$\Pi_F^D(Q) = 0. \quad (2.11)$$

Here it is worth making the following remark. As in [7], \mathcal{L}_0 in (2.1) - (2.4) is non-hermitian, since Σ_F , $\Pi_F^{\mu\nu}$, and Π_F^g are complex functions. Through a standard procedure, the quasifree Hamiltonian, H_0 , is constructed from (2.1) - (2.4), which is also non-hermitian. We recall that, in constructing the Gell-Mann-Low formula of perturbation theory in vacuum theory, the hermiticity of the free Hamiltonian plays an essential role. In the operator formalism of thermal field theory, which is called thermo field dynamics [7], the so-called hat-Hamiltonian, \hat{H} , plays the role of Hamiltonian, H , in vacuum theory. \hat{H} is defined as $\hat{H} = H - \tilde{H}$, where \tilde{H} is

constructed from H through the so-called tilde-conjugation rules. The Gell-Mann-Low formula may be derived [7] by choosing a free Hamiltonian, H_0 , from which the hat-Hamiltonian, $\hat{H}_0 = H_0 - \tilde{H}_0$, is constructed. It should be stressed that H_0 is not necessarily hermitian. In the course of derivation, the so-called tildicity of \hat{H}_0 , i.e., the invariance of $-i\hat{H}_0$ under the tilde conjugation, plays the role of hermiticity of H_0 in vacuum theory. It is well known that, as far as thermal-equilibrium cases are concerned, both the above operator formalism and the conventional real-time thermal field theory (constructed on a time-path in a complex time plane) lead to the same Feynman rules in perturbative calculation.

The interaction Lagrangian density is defined as

$$\mathcal{L}_{\text{int}} = \mathcal{L}_{\text{QCD}} - \mathcal{L}_0. \quad (2.12)$$

On the basis of the theory defined by (2.1) - (2.4) and (2.12), we shall determine $\Sigma_F(Q)$, $\Pi_F^{\mu\nu}(Q)$, and $\Pi_F^g(Q)$ self consistently, to leading order at logarithmic accuracy. We employ the closed-time-path formalism of real-time thermal field theory [9].

The diagonalized or Feynman propagator of the quark constructed from (2.2) and (2.5) is

$${}^\diamond S_F(Q) = -\frac{1}{2} \sum_{\tau=\pm} \hat{\phi}_\tau \frac{1}{{}^\diamond D_\tau(Q)},$$

where

$$\begin{aligned} \hat{Q}_\tau^\mu &\equiv (1, \tau \hat{\mathbf{q}}), \\ {}^\diamond D_\tau(Q) &= (-q_0 + \tau q) \{1 - f(Q)\} + g(Q). \end{aligned} \quad (2.13)$$

Each component of the 2×2 matrix propagator is obtained from ${}^\diamond S_F(Q)$ through Bogoliubov transformation [9]:

$${}^\diamond S^{(ji)}(Q) = \sum_{\tau=\pm} \hat{\phi}_\tau {}^\diamond \tilde{S}_\tau^{(ji)}(Q), \quad (j, i = 1, 2), \quad (2.14)$$

where i and j are the thermal indexes and

$$\begin{aligned} \text{Re } {}^\diamond \tilde{S}_\tau^{(11)}(Q) &= -\text{Re } {}^\diamond \tilde{S}_\tau^{(22)}(Q) \\ &= -\frac{1}{2} \text{Re } \frac{1}{{}^\diamond D_\tau(Q)}, \end{aligned} \quad (2.15)$$

$$\begin{aligned} \text{Im } {}^\diamond \tilde{S}_\tau^{(11)}(Q) &= \text{Im } {}^\diamond \tilde{S}_\tau^{(22)}(Q) \\ &= -\pi \epsilon(q_0) \left(\frac{1}{2} - n_F(|q_0|) \right) {}^\diamond \rho_\tau(Q), \end{aligned} \quad (2.16)$$

$${}^\diamond \tilde{S}_\tau^{(12)/(21)}(Q) = \pm i\pi n_F(\pm q_0) {}^\diamond \rho_\tau(Q). \quad (2.17)$$

Here $\epsilon(q_0) \equiv q_0/|q_0|$, $n_F(x) \equiv 1/(e^{x/T} + 1)$, and

$${}^\diamond \rho_\tau(Q) = \frac{\epsilon(q_0)}{\pi} \text{Im} \frac{1}{{}^\diamond D_\tau(Q)}. \quad (2.18)$$

The diagonalized FP-ghost propagator ${}^\diamond \Delta_F^g(Q)$ obtained from (2.4) is

$${}^\diamond \Delta_F^g(Q) = \frac{1}{Q^2 - \Pi_F^g(Q)}. \quad (2.19)$$

Using (2.3) and (2.6) - (2.10), we obtain [9, 10] for the diagonalized gluon propagator, ${}^\diamond \Delta_F^{\mu\nu}(Q)$:

$$\begin{aligned} {}^\diamond \Delta_F^{\mu\nu}(Q) &= -\mathcal{P}_T^{\mu\nu}(Q) {}^\diamond \Delta_F^T(Q) - \mathcal{P}_L^{\mu\nu}(Q) {}^\diamond \Delta_F^L(Q) \\ &\quad - \mathcal{C}^{\mu\nu}(Q) {}^\diamond \Delta_F^C(Q) - \mathcal{D}^{\mu\nu}(Q) {}^\diamond \Delta_F^D(Q), \end{aligned} \quad (2.20)$$

$${}^\diamond \Delta_F^T(Q) = \frac{1}{Q^2 - \Pi_F^T(Q)}, \quad (2.21)$$

$${}^\diamond \Delta_F^L(Q) = \frac{1}{Q^2 - \Pi_F^L(Q)}, \quad (2.22)$$

$${}^\diamond \Delta_F^C(Q) = \eta \frac{\Pi_F^C(Q)}{(Q^2 - \Pi_F^L(Q))(Q^2 + i0^+)}, \quad (2.23)$$

$${}^\diamond \Delta_F^D(Q) = \eta \frac{1}{Q^2 + i0^+}, \quad (2.24)$$

where use has been made of (2.11). 2×2 FP-ghost- and gluon-propagators are obtained, respectively, from ${}^\diamond \Delta_F^g(Q)$ and ${}^\diamond \Delta_F^{\mu\nu}(Q)$ through Bogoliubov transformation [9]:

$$\begin{aligned} {}^\diamond \Delta^{g/\mu\nu(11)}(Q) &= -\left[{}^\diamond \Delta^{g/\mu\nu(22)}(Q)\right]^* \\ &= [1 + n_B(|q_0|)] {}^\diamond \Delta_F^{g/\mu\nu}(Q) - n_B(|q_0|) \left({}^\diamond \Delta_F^{g/\mu\nu}(Q)\right)^*, \end{aligned} \quad (2.25)$$

$${}^\diamond \Delta^{g/\mu\nu(12)}(Q) = [\theta(-q_0) + n_B(|q_0|)] \left[{}^\diamond \Delta_F^{g/\mu\nu}(Q) - \left({}^\diamond \Delta_F^{g/\mu\nu}(Q)\right)^*\right], \quad (2.26)$$

$${}^\diamond \Delta^{g/\mu\nu(21)}(Q) = [\theta(q_0) + n_B(|q_0|)] \left[{}^\diamond \Delta_F^{g/\mu\nu}(Q) - \left({}^\diamond \Delta_F^{g/\mu\nu}(Q)\right)^*\right]. \quad (2.27)$$

Here we recall that the invariance of \mathcal{L}_{QCD} under the BRS transformation leads to the Ward-Takahashi relation [9],

$$\begin{aligned} Q^\nu \Delta_{\mu\nu}'^{ab(rs)}(Q) &= -\eta \left[Q_\mu \Delta_g'^{ab(rs)}(Q) \right. \\ &\quad \left. - (-)^{r-1} \Pi_{g;\mu}'^{ac(rt)}(Q) \Delta_g'^{cb(ts)}(Q) \right], \end{aligned} \quad (2.28)$$

where r, s, t are thermal indexes, a, b, c are color indexes, and $\Delta'^{ab}_s (\equiv \delta^{ab} \Delta'_s)$ are full propagators. $\Pi'_{g;\mu}{}^{ac}(rt)(Q)$ is the pre-self-energy-part for a FP-ghost, which satisfies $Q^\mu \Pi'_{g;\mu}{}^{ac}(rt)(Q) = \Pi'^{ac}(rt)(Q)$.

Multiplying Q^μ to the both sides of (2.28) and using the Schwinger-Dyson equation for $\Delta'^{ab(rs)}(Q)$, one obtains [9]

$$\Delta'^{D(rs)}(Q) = \eta \Delta_g^{(0)(rs)}(Q), \quad (2.29)$$

where $\Delta_g^{(0)(rs)}(Q)$ is the bare FP-ghost propagator. Eq. (2.29) leads to (2.11).

Projecting out $\Delta'^{C(rs)}(Q)$, which is defined as in (2.20), from (2.28), we obtain

$$\begin{aligned} \Delta'^{C(rs)}(Q) = & \eta \frac{\sqrt{2}q}{q_0} \left[\Delta'_g{}^{(rs)}(Q) - \Delta_g^{(0)(rs)}(Q) \right. \\ & \left. + (-)^{r-1} \frac{1}{q^2} \tilde{Q}_\mu \Pi'_{g;\mu}{}^{ab}(rt)(Q) \Delta'_g{}^{ba(ts)}(Q) \right], \end{aligned}$$

where the sum is not taken over a in the last term, which is independent of a , thanks to the $SU(N)$ symmetry. As will be seen in Sec. V, the region of our interest is $|Q^2| \ll q^2 = O(T^2)$, where the first and second terms in the square brackets are of $O(1/Q^2)$, while the third term is of $O(1)$. Neglecting the third term, we obtain for the leading contribution to the diagonalized propagator, ${}^\diamond\Delta_F^C(Q)$,

$${}^\diamond\Delta_F^C(Q) \simeq \eta \frac{\sqrt{2}q}{q_0} \frac{\Pi_F^g(Q)}{(Q^2 - \Pi_F^g(Q))(Q^2 + i0^+)}, \quad (2.30)$$

where $|q_0| \simeq q$. Comparison of (2.23) with (2.30) yields

$$\Pi_F^L(Q) \simeq \Pi_F^g(Q), \quad (2.31)$$

$$\Pi_F^C(Q) \simeq \sqrt{2} \epsilon(q_0) \Pi_F^L(Q), \quad (2.32)$$

which are valid at $|Q^2| \ll q^2$. We choose Π_F s in (2.3) and (2.4) so as to satisfy the relations (2.11), (2.31), and (2.32).

3 Hard-quark mode

In this section, we determine Σ_F in (2.2) self-consistently to one loop-order. The diagram to be analyzed is depicted in Fig. 1. In Fig. 1(a), for soft K [$Q - K$], the

HTL-resummed effective gluon [quark] propagator ${}^*\Delta^{(ji)}(K)$ [${}^*S^{(ji)}(Q-K)$] should be assigned. It is to be noted that in calculating ${}^*\Delta^{(ji)}(K)$ and ${}^*S^{(ji)}(Q-K)$ the forms ${}^\circ S^{(ji)}$ and ${}^\circ \Delta^{(ji)}$, obtained in Sec. II, should be used for, respectively, hard-quark- and hard-gluon-propagators in the HTL.

Let $\tilde{\Sigma}_F^{(1a)}$ be the contribution of Fig. 1(a). A dimensional analysis shows that, for $|Q^2| \gg g^2 T^2$, $|\tilde{\Sigma}_F^{(1a)}(Q)| \ll |Q^2|/q$ and, up to a possible factor of $\ln(g^{-1})$, $\tilde{\Sigma}_F^{(1a)} = O(g^2 T)$ for $|Q^2| \leq O(g^2 T^2)$. Then it is sufficient to analyze Fig. 1 in the region

$$||q_0| - q| \leq O(g^2 T). \quad (3.1)$$

The contribution from Fig. 1(b), $\tilde{\Sigma}_F^{(1b)}$, is obtained from (2.2) and (2.12):

$$\tilde{\Sigma}_F^{(1b)}(Q) = -\Sigma_F(Q). \quad (3.2)$$

Computation of Fig. 1(a) in conventional hot QCD is carried out e.g. in [11], where it has been shown that, to leading order,

$$Re \tilde{\Sigma}_F^{(1a)}(Q) \Big|_{\text{conventional}} \simeq m_f^2 \frac{q_0}{q^2} \gamma^0. \quad (3.3)$$

Here $m_f^2 = g^2 C_F T^2 / 8$ with $C_F = (N^2 - 1)/(2N)$. The leading contribution (3.3), being gauge independent, comes from the region where K and $Q - K$ in Fig. 1(a) are hard. In other ward, the result (3.3) is insensitive to the soft- K and soft- $(Q - K)$ region in Fig 1(a). The difference between the thermal propagator ${}^\circ S^{(ji)}(Q)$ [${}^\circ \Delta^{(ji)}(Q)$] constructed in Sec. II and the one $S^{(ji)}(Q)$ [$\Delta^{(ji)}(Q)$] in conventional hot QCD cannot be ignored at the region $|Q^2| \leq O(g^2 T^2)$. The result (3.3) is however insensitive to the region $|K^2|, |(Q - K)^2| \leq O(g^2 T^2)$, i.e., Fig. 1(a) with ${}^\circ S^{(ji)}(Q - K)$ and ${}^\circ \Delta^{(ji)}(K)$ for, respectively, the bare hard-quark and hard-gluon propagators yields, to leading order, the same result (3.3):

$$Re \tilde{\Sigma}_F^{(1a)}(Q) \simeq m_f^2 \frac{q_0}{q^2} \gamma^0. \quad (3.4)$$

Now we impose the self-consistency condition,

$$\tilde{\Sigma}_F(Q) = \tilde{\Sigma}_F^{(1a)}(Q) + \tilde{\Sigma}_F^{(1b)}(Q) = 0,$$

or

$$\Sigma_F(Q) = \tilde{\Sigma}_F^{(1a)}(Q), \quad (3.5)$$

where use has been made of (3.2). Substituting (3.4) into (3.5), we obtain

$$Re \Sigma_F(Q) \simeq m_f^2 \frac{q_0}{q^2} \gamma^0$$

or, from (2.5),

$$\begin{aligned} Re f(Q) &\simeq 0, \\ Re g(Q) &\simeq m_f^2 \frac{q_0}{q^2}. \end{aligned} \quad (3.6)$$

Here $Re f(Q) \simeq 0$ means $|Re f(Q)| \ll g^2 T$.

We are now in a position to compute $Im \tilde{\Sigma}_F^{(1a)}$. Shown in [11] is that, in conventional hot QCD calculation, the leading contribution (at logarithmic accuracy) to $Im \tilde{\Sigma}_F^{(1a)} \Big|_{\text{conventional}}$ comes from Fig. 1(a) with soft K . More precisely, the magnetic part of the soft-gluon propagator yields the leading contribution. In other words, the contributions from the electric part and from the gauge-parameter-dependent part of the soft-gluon propagator are nonleading. This means in particular that the leading contribution is gauge independent. In contrast to the case of $Re \tilde{\Sigma}_F^{(1a)}$, $Im \tilde{\Sigma}_F^{(1a)}$ is logarithmically sensitive to the region $(Q - K)^2 \simeq 0$, so that we should compute $Im \tilde{\Sigma}_F^{(1a)}$ in the theory defined in Sec II.

As has been mentioned above, we are interested in the region (3.1). In this region, $|q_0| (\equiv \tau q_0) \simeq q$, we write $f(Q)$ and $g(Q)$ in (2.5)

$$f_\tau(q_0, q) \equiv f(Q), \quad g_\tau(q_0, q) \equiv g(Q), \quad (3.7)$$

which satisfies $f_\tau(q_0, q) = f_{-\tau}(-q_0, q)$ and $g_\tau(q_0, q) = -g_{-\tau}(-q_0, q)$.

Leading contribution to $Im \tilde{\Sigma}_F^{(1a)}$ reads [9],

$$\begin{aligned} Im \tilde{\Sigma}_F^{(1a)}(Q) &= \frac{i}{2[\theta(q_0) - n_F(q)]} \tilde{\Sigma}_{(1a)}^{(21)}(Q) \\ &\simeq \frac{g^2 C_F}{2[\theta(q_0) - n_F(q)]} \int_{\text{soft } K} \frac{d^4 K}{(2\pi)^4} \gamma_\rho \\ &\quad \times {}^\diamond S^{(21)}(Q - K) \gamma_\sigma {}^* \Delta_{\rho\sigma}^{\text{mag}(21)}(K), \end{aligned} \quad (3.8)$$

where ${}^* \Delta_{\rho\sigma}^{\text{mag}(21)}$ is the $(2, 1)$ -component of the magnetic part of the effective gluon propagator [12],

$${}^* \Delta_{\rho\sigma}^{\text{mag}(21)}(K) \equiv -\mathcal{P}_{T\rho\sigma}(K) {}^* \tilde{\Delta}^{\text{mag}(21)}(K), \quad (3.9)$$

$$\begin{aligned} {}^* \tilde{\Delta}^{\text{mag}(21)}(K) &\simeq {}^* \tilde{\Delta}^{\text{mag}(ij)}(K) \simeq -2\pi i \frac{T}{k_0} \rho_t(K), \\ &\quad (i, j = 1, 2). \end{aligned} \quad (3.10)$$

Here $\mathcal{P}_{T\rho\sigma}(K)$ is as in (2.7) and $\rho_t(K)$ is the spectral function. As has been mentioned at the beginning of this section, in computing ${}^*\Delta_{\rho\sigma}^{\text{mag}(21)}(K)$, we should use ${}^\circ S^{(ji)}$ and ${}^\circ \Delta^{(ji)}$ for hard propagators in the HTL. It is not difficult to see that the result (3.18) below is not sensitive to this “modification” of the hard propagators. Then, in the following, we shall use the form of ${}^*\Delta_{\rho\sigma}^{\text{mag}(21)}(K)$, computed in conventional hot QCD [12].

Substituting (2.14), (3.9), and (3.10) into (3.8), we obtain

$$\begin{aligned} \text{Im } \tilde{\Sigma}_F^{(1a)}(Q) &\simeq -\frac{ig^2 C_F T}{\theta(q_0) - n_F(q)} \int_{\text{soft } K} \frac{d^4 K}{(2\pi)^3} \frac{1}{k_0} \rho_t(K) \\ &\times \sum_{\tau=\pm} \left[\hat{\mathcal{Q}}_\tau + \tau \hat{\mathbf{q}} \cdot \vec{\gamma} - \tau (\vec{\gamma} \cdot \hat{\mathbf{k}}) (\hat{\mathbf{k}} \cdot \hat{\mathbf{q}}) \right] \\ &\times {}^\circ \tilde{S}_\tau^{(21)}(Q - K). \end{aligned} \quad (3.11)$$

${}^\circ \tilde{S}_\tau^{(21)}(P)$ with $P = Q - K$ is obtained from (2.17) with (2.18), (2.13), and (3.7):

$$\begin{aligned} {}^\circ \tilde{S}_\tau^{(21)}(P) &\simeq [\theta(p_0) - n_F(p)] \\ &\times \frac{g_\tau^{(i)} + (p_0 - \tau p) f_\tau^{(i)}}{[(p_0 - \tau p) \{1 - f_\tau^{(r)}\} - g_\tau^{(r)}]^2 + [g_\tau^{(i)} + (p_0 - \tau p) f_\tau^{(i)}]^2}, \end{aligned}$$

where $f_\tau^{(r)} \equiv \text{Re } f_\tau^{(r)}(p_0, p)$, $f_\tau^{(i)} = \text{Im } f_\tau^{(i)}(p_0, p)$, etc. Using (3.6) with (3.7), we have

$${}^\circ \tilde{S}_\tau^{(21)}(Q - K) \simeq i [\theta(q_0) - n_F(q)] \frac{\tilde{g}_\tau^{(i)}(q_0, q)}{[q_0 - \tau(q + m_f^2/q) - k_0 + \tau \mathbf{k} \cdot \hat{\mathbf{q}}]^2 + [\tilde{g}_\tau^{(i)}(q_0, q)]^2}, \quad (3.12)$$

where Q is hard, K is soft, and

$$\tilde{g}_\tau^{(i)}(q_0, q) \equiv g_\tau^{(i)}(q_0, q) + (q_0 - \tau q) f_\tau^{(i)}(q_0, q). \quad (3.13)$$

We shall show below (cf. (3.19) with (2.5)) that the second term on the R.H.S. of (3.13) turns out to be negligible when compared to the first term, so that

$$\tilde{g}_\tau^{(i)}(q_0, q) \simeq g_\tau^{(i)}(q_0, q).$$

Substituting (3.12) into (3.11), we obtain

$$\begin{aligned} \text{Im } \tilde{\Sigma}_F^{(1a)}(Q) &\simeq g^2 C_F T \int_{\text{soft } K} \frac{d^4 K}{(2\pi)^3} \frac{\rho_t(K)}{k_0} \left[\hat{\mathcal{Q}}_\tau + \tau \hat{\mathbf{q}} \cdot \vec{\gamma} - \tau (\vec{\gamma} \cdot \hat{\mathbf{k}}) (\hat{\mathbf{k}} \cdot \hat{\mathbf{q}}) \right] \\ &\times \frac{g_\tau^{(i)}(q_0, q)}{[q_0 - \tau(q + m_f^2/q) - k_0 + \tau \mathbf{k} \cdot \hat{\mathbf{q}}]^2 + [g_\tau^{(i)}(q_0, q)]^2}. \end{aligned} \quad (3.14)$$

It is well known [3, 4, 5] that, at logarithmic accuracy, the dominant contribution comes from the region where $|k_0| \ll k$ and $|\hat{\mathbf{k}} \cdot \hat{\mathbf{q}}| \ll 1$. Then, the piece $-\tau(\vec{\gamma} \cdot \hat{\mathbf{k}})(\hat{\mathbf{k}} \cdot \hat{\mathbf{q}})$ in (3.14) leads to a nonleading contribution. Setting $k_0 = 0$ in the denominator of the last term in (3.14) and integrating over $-\alpha < \hat{\mathbf{k}} \cdot \hat{\mathbf{q}} < +\alpha$ with $\alpha \ll 1$, we obtain

$$Im \tilde{\Sigma}_F^{(1a)}(Q) \simeq \frac{g^2}{4\pi^2} C_F T \gamma^0 \int_{\text{soft } K} dk k \int \frac{dk_0}{k_0} \rho_t(K) \frac{g_i}{|g_i|} \times \sum_{\xi=\pm} \arctan \left(\frac{\alpha k + \xi \{q_0 - \tau(q + m_f^2/q)\}}{|g_\tau^{(i)}(q_0, q)|} \right). \quad (3.15)$$

As mentioned at the beginning of this section, up to a possible factor of $\ln(g^{-1})$, $|g_\tau^{(i)}(q_0, q)|$ is of $O(g^2 T)$ and we are interested in the region $|q_0 - \tau(q + m_f^2/q)| = O(g^2 T)$ (cf. (3.1)). At the region $|k_0| \ll k$,

$$\rho_t(K) \simeq \mathcal{M}_T^2 \frac{k k_0}{k^6 + (\pi \mathcal{M}_T^2)^2 k_0^2}, \quad (3.16)$$

where $\mathcal{M}_T^2 \equiv 3m_T^2/4$ with

$$m_T^2 = \frac{1}{9} \left(N + \frac{N_f}{2} \right) (gT)^2. \quad (3.17)$$

Substituting (3.16) into (3.15) and integrating over $-\beta k < k_0 < +\beta k$ with $\beta \ll 1$, we get

$$Im \tilde{\Sigma}_F^{(1a)}(Q) \simeq \frac{2}{\pi^2} \frac{g^2}{4\pi} C_F T \gamma^0 \int_{\text{soft } k} \frac{dk}{k} \arctan \left(\frac{\pi \beta \mathcal{M}_T^2}{k^2} \right) \frac{g_\tau^{(i)}(q_0, q)}{|g_\tau^{(i)}(q_0, q)|} \times \sum_{\xi=\pm} \arctan \left(\frac{\alpha k + \xi \{q_0 - \tau(q + m_f^2/q)\}}{|g_\tau^{(i)}(q_0, q)|} \right).$$

Now we observe that $\arctan\{\pi\beta\mathcal{M}_T^2/k^2\} = \pi/2$ at $k = 0$ and $\propto \beta m_T^2/k^2 \propto (gT/k)^2$ for $k \gg gT$. The transition region is $k = O(gT)$. The quantity at the second line vanishes at $k = 0$ and $\simeq \pi$ for $k \gg |g_\tau^{(i)}|$. When $O\{|q_0 - \tau(q + m_f^2/q)|\} \leq O(|g_\tau^{(i)}|)$, the transition region is $k = O(|g_\tau^{(i)}|)$ and, when $O\{|q_0 - \tau(q + m_f^2/q)|\} > O(|g_\tau^{(i)}|)$, the transition region is $k = O\{|q_0 - \tau(q + m_f^2/q)|\}$.

From the above observation, we obtain for the leading contribution at logarithmic accuracy,

$$Im \tilde{\Sigma}_F^{(1a)}(Q) \simeq \frac{g^2}{4\pi} C_F T \gamma^0 \frac{g_\tau^{(i)}(q_0, q)}{|g_\tau^{(i)}(q_0, q)|} \ln \left(\frac{m_T}{\Gamma_q(Q)} \right), \quad (3.18)$$

where

$$\Gamma_q(Q) \equiv \max \left[|q_0 - \tau(q + m_f^2/q)|, |g_\tau^{(i)}(q_0, q)| \right] .$$

It should be emphasized again that (3.18) is valid at logarithmic accuracy, i.e., the term of $O(1)$ is ignored when compared to $\ln\{m_T/\Gamma_q(Q)\}$.

Substituting (3.18) into the self-consistency condition (3.5), we obtain

$$Im \Sigma_F(Q) \simeq \frac{g^2}{4\pi} C_F T \gamma^0 \frac{g_\tau^{(i)}(q_0, q)}{|g_\tau^{(i)}(q_0, q)|} \ln \left(\frac{m_T}{\Gamma_q(Q)} \right) . \quad (3.19)$$

From (2.5), (3.7), and (3.19), we have

$$\begin{aligned} f_\tau^{(i)}(q_0, q) &\simeq 0 , \\ g_\tau^{(i)}(q_0, q) &= \frac{g^2}{4\pi} C_F T \frac{g_\tau^{(i)}(q_0, q)}{|g_\tau^{(i)}(q_0, q)|} \ln \left(\frac{m_T}{\max \left[|q_0 - \tau(q + m_f^2/q)|, |g_\tau^{(i)}(q_0, q)| \right]} \right) . \end{aligned} \quad (3.20)$$

$$(3.21)$$

It is to be noted that, in conventional hot QCD, we have [11] (3.18) with $g_\tau^{(i)}(q_0, q)/|g_\tau^{(i)}(q_0, q)| = -\epsilon(q_0)$ and $\Gamma_q(Q) = |q_0 - \tau(q + m_f^2/q)|$. We also note that, for $q_0 > 0$,

$$\begin{aligned} -i \epsilon(q_0) \text{tr} \left[\not{Q} \tilde{\Sigma}_{(1a)}^{(21)}(Q) \right] &= -2\epsilon(q_0) [\theta(q_0) - n_F(q)] \\ &\times \text{tr} \left[\not{Q} Im \tilde{\Sigma}_F^{(1a)}(Q) \right] , \end{aligned} \quad (3.22)$$

with $\tilde{\Sigma}_{(1a)}^{(21)}$ as in (3.8), is proportional to the decay rate of a quark mode, whose propagator is given by (2.14) - (2.18). While for $q_0 < 0$, (3.22) is proportional to the production rate. Then (3.22) should be positive, which means again that $g_\tau^{(i)}(q_0, q)/|g_\tau^{(i)}(q_0, q)| = -\epsilon(q_0)$. From these observations, as the physically sensible solution, we assume that $g_\tau^{(i)}(q_0, q)/|g_\tau^{(i)}(q_0, q)| = -\epsilon(q_0)$.

As has been mentioned repeatedly, we see from (3.21) that the contribution of $O\{g^2 T \ln(g^{-1})\}$ to $g_\tau^{(i)}$ emerges in the region,

$$||q_0| - q| \leq O\{g^2 T \ln(g^{-1})\} . \quad (3.23)$$

In fact, by taking the logarithm of (3.21), we can solve the resulting equation iteratively with respect to $\ln |g_\tau^{(i)}(q_0, q)|$ to obtain

$$g_\tau^{(i)}(q_0, q) \equiv -\epsilon(q_0) \gamma_q$$

$$\begin{aligned}\gamma_q &= \frac{g^2}{4\pi} C_F T \ln(g^{-1}) \left[1 - \frac{\ln\{\ln(g^{-1})\}}{\ln(g^{-1})} + \mathcal{F} \right] \\ &\quad + O(g^2 T). \end{aligned} \quad (3.24)$$

When $||q_0| - q| = O\{g^2 T \ln(g^{-1})\}$, $\mathcal{F} = 0$, while for $||q_0| - q| < O\{g^2 T \ln(g^{-1})\}$, $\mathcal{G} \equiv 1 - \ln\{\ln(g^{-1})\}/\ln(g^{-1}) + \mathcal{F}$ is determined through $\mathcal{G} = 1 - \ln\{\mathcal{G} \ln(g^{-1})\}/\ln(g^{-1})$.

Here we summarize the results obtained above. From (2.5) with (3.7), (3.6), (3.20), and (3.24), we have for the self-consistently determined $\Sigma_F(Q)$,

$$\Sigma_F(Q) \simeq \tau \left[\frac{m_f^2}{q} - i\gamma_q(Q) \right] \gamma^0,$$

where $\tau = \epsilon(q_0)$. It should be mentioned that we have evaluated γ_q at logarithmic accuracy. Namely, the computation of the $O(g^2 T)$ contribution to γ_q , Eq. (3.24), is outside the scope of this paper. Taking this fact into account, we obtain from (2.13) with (3.7), (3.6), (3.20), and (3.24),

$$\begin{aligned} \text{Re} \frac{1}{\circ D_\tau(q_0, q)} &\simeq - \frac{q_0 - \tau(q + m_f^2/q)}{[q_0 - \tau(q + m_f^2/q)]^2 + \gamma_q^2} \\ &\simeq - \frac{q_0 - \tau(q + m_f^2/q)}{(q_0 - \tau q)^2 + \gamma_q^2}, \\ \text{Im} \frac{1}{\circ D_\tau(q_0, q)} &\simeq \frac{\tau \gamma_q}{(q_0 - \tau q)^2 + \gamma_q^2}. \end{aligned}$$

Thus the resummation of the imaginary-part of the self-energy part, $\text{Im} \tilde{\Sigma}_F$, plays a dominant role.

Then, the thermal propagator of the “good” mode with hard momentum Q reads (cf. (2.14) - (2.18))

$$\circ S^{(ji)}(Q) = \sum_{\tau=\pm} \hat{\phi}_\tau \circ \tilde{S}_\tau^{(ji)}(Q), \quad (j, i = 1, 2), \quad (3.25)$$

$$\begin{aligned} \text{Re} \circ \tilde{S}_\tau^{(11)}(Q) &= -\text{Re} \circ \tilde{S}_\tau^{(22)}(Q) \\ &\simeq \frac{1}{2} \frac{q_0 - \epsilon(q_0)(q + m_f^2/q)}{(q_0 - \tau q)^2 + \gamma_q^2}, \end{aligned} \quad (3.26)$$

$$\begin{aligned} \text{Im} \circ \tilde{S}_\tau^{(11)}(Q) &= \text{Im} \circ \tilde{S}_\tau^{(22)}(Q) \\ &\simeq -\pi \epsilon(q_0) \left[\frac{1}{2} - n_F(q) \right] \circ \rho_\tau(Q), \end{aligned} \quad (3.27)$$

$$\circ \tilde{S}_\tau^{(12)/(21)}(Q) \simeq -i\pi \epsilon(q_0) [\theta(\mp q_0) - n_F(q)] \circ \rho_\tau(Q), \quad (3.28)$$

where

$$\circ\rho_\tau(Q) \simeq \frac{1}{\pi} \frac{\gamma_q}{(q_0 - \tau q)^2 + \gamma_q^2}. \quad (3.29)$$

The forms (3.27) and (3.28) are valid in the region (3.23), while the form (3.26) is valid in the region $O(g^3T) < |q_0 - \epsilon(q_0)(q + m_f^2/q)| \leq O\{g^2T \ln(g^{-1})\}$. For obtaining $Re \circ\tilde{S}_\tau^{(11)}(Q)$ in the region $|q_0 - \epsilon(q_0)(q + m_f^2/q)| \leq O(g^3T)$, concrete evaluation of Fig. 1(a) as well as the two-loop contribution is necessary.

4 Absence of additional contributions of leading order

In this section, we analyze some other formally higher-order corrections to the hard-quark self-energy part and show that they are nonleading.

4.1 Analysis of Figs. 2 - 5

As has been recognized from the analysis in Sec. III, in conventional hot QCD, resummation of the one-loop self-energy part should be carried out for a thermal propagator of a hard quark close to the mass shell, $||q_0| - q| \leq O\{g^2T \ln(g^{-1})\}$. It was shown, e.g., in [11] that the same “phenomenon” occurs in the case of quark-gluon vertex. An one-loop contribution to the quark-gluon vertex is depicted in Fig. 2, where K is soft and Q is hard. When Q and $Q - K$ are close to the mass shell, $|Q^2|, |(Q - K)^2| \leq O\{g^2T^2 \ln(g^{-1})\}$, the contribution of Fig. 2 is of the same order of magnitude as the bare counterpart. As in the self-energy case, the leading contribution comes from the magnetic part of the soft-gluon propagator in Fig. 2 and thus is gauge independent. The same is true for multi-loop contributions.

Fig. 2 yields

$$\begin{aligned} \left(\Lambda_{g1}^{a;\mu}(Q - K, Q)\right)_{ji}^\ell &\simeq ig^2(-)^{i+j+\ell} \left(\frac{N}{2} - C_F\right) T^a \\ &\times \int_{\text{soft } P} \frac{d^4P}{(2\pi)^4} \gamma^\sigma \circ S^{(j\ell)}(Q - K + P) \gamma^\mu \\ &\times \circ S^{(\ell i)}(Q + P) \gamma^\rho * \Delta_{\rho\sigma}^{\text{mag}(ij)}(P), \end{aligned} \quad (4.1)$$

where T^a are the (hermitian) fundamental-representation matrix of $su(N)$. Substituting (3.9), (3.10), and (3.25), we obtain, after some manipulation,

$$\begin{aligned} \left(\Lambda_{g1}^{a;\mu}(Q-K, Q) \right)_{ji}^\ell &\simeq 4g^2(-)^{i+j+\ell} \left(\frac{N}{2} - C_F \right) T^a T \\ &\times \sum_{\tau=\pm} \hat{Q}_\tau \int_{\text{soft } P} \frac{d^4 P}{(2\pi)^3} \left[\hat{\mathcal{Q}}_\tau + \tau \hat{\mathbf{q}} \cdot \vec{\gamma} - \tau (\vec{\gamma} \cdot \hat{\mathbf{p}})(\hat{\mathbf{p}} \cdot \hat{\mathbf{q}}) \right] \\ &\times \frac{\rho_t(P)}{p_0} \circ \tilde{S}_\tau^{(j\ell)}(Q-K+P) \circ \tilde{S}_\tau^{(\ell i)}(Q+P). \end{aligned}$$

As will be shown in Appendix A, the dominant contribution comes from the region where $|\hat{\mathbf{p}} \cdot \hat{\mathbf{q}}| \ll 1$, so that

$$\begin{aligned} \hat{\mathcal{Q}}_\tau + \tau \hat{\mathbf{q}} \cdot \vec{\gamma} - \tau (\vec{\gamma} \cdot \hat{\mathbf{p}})(\hat{\mathbf{p}} \cdot \hat{\mathbf{q}}) \\ \simeq \hat{\mathcal{Q}}_\tau + \tau \hat{\mathbf{q}} \cdot \vec{\gamma} \\ = \gamma^0. \end{aligned}$$

Then, we have

$$\begin{aligned} \left(\Lambda_{g1}^{a;\mu}(Q-K, Q) \right)_{ji}^\ell &\simeq \frac{g^2}{\pi^2} (-)^{i+j+\ell} \left(\frac{N}{2} - C_F \right) T^a T \gamma^0 \\ &\times \sum_{\tau=\pm} \hat{Q}_\tau^\mu \mathcal{S}_\tau^{j\ell i}, \end{aligned} \tag{4.2}$$

where

$$\begin{aligned} \mathcal{S}_\tau^{ijk\ell} &\equiv \int dp p^2 \int \frac{dp_0}{p_0} \rho_t(P) \\ &\times \int dz \circ \tilde{S}_\tau^{(ij)}(Q-K+P) \circ \tilde{S}_\tau^{(k\ell)}(Q+P). \end{aligned}$$

Obviously $\tau = q_0/|q_0|$ sector yields the leading contribution. \mathcal{S}_τ s are computed in Appendix A. From (A.4) in Appendix A, we see that, in the region $|K \cdot \hat{Q}_\tau| = O(\Gamma_q)$, \mathcal{S}_τ s are of $O(L/\Gamma_q) = O\{1/(g^2 T)\}$ where L is as in (A.3). Then $\left(\Lambda_{g1}^{a;\mu}(Q-K, Q) \right)_{ji}^\ell$ in (4.2) is of $O(1)$, the same order of magnitude as the lowest-order counterpart, $(-)^{\ell-1} \delta_{\ell j} \delta_{\ell i} T^a \gamma^\mu$. It is worth noting that, as in the self-energy case dealt with in Sec. III, the dominant contribution to \mathcal{S}_τ s comes from the region where $|p_0| \ll p$ (cf. Appendix A).

Now we substitute Fig. 2 for the quark-gluon vertex on the left side of Fig. 1(a) to obtain Fig. 3. Figure 3 consists of four contributions corresponding to $(i, j) =$

(1, 1), (1, 2), (2, 1), and (2, 2). We first note that the form of the leading part of the magnetic soft gluon propagator ${}^*\Delta_{\rho\sigma}^{\text{mag}(ij)}(P)$ is independent of the thermal indexes i and j (cf. (3.10)). Then Fig. 3 contains \mathcal{S}_τ s in the combination $\sum_{i=1}^2 (-)^i \mathcal{S}_\tau^{jii1}$. From (A.4) and (A.5) in Appendix A, we see that the cancellation occurs in the above combination,

$$\sum_{i=1}^2 (-)^i \mathcal{S}_\tau^{jii1} \simeq 0.$$

This means that, although each of the four contributions of Fig. 3 is of the same order of magnitude as the contribution of Fig. 1, cancellations occur between them and the contribution of Fig. 3 turns out to be nonleading.

Now let us turn to analyze multi-loop contributions. We first inspect the ladder diagram as depicted in Fig. 4, where solid- and dashed-lines stand, respectively, for quark- and gluon-propagators, Q is hard, and P_j s are soft. We are interested in the behavior at $K \cdot \hat{Q}_\tau \simeq 0$. As will be shown below, the contribution from the region where P_j ($1 \leq j \leq n$) are soft is of the same order of magnitude as the lowest-order counterpart, $-(-)^\ell \delta_{\ell j_1} \delta_{i_1} T^a \gamma^\mu$. In place of (4.1), we have

$$\begin{aligned} & \left(\Lambda_{gn}^{a;\mu}(Q - K, Q) \right)_{j_1 i_1}^\ell \\ &= -(-)^\ell \left[-ig^2 \left(\frac{N}{2} - C_F \right) \right]^n T^a \\ & \times \sum_{i_2, \dots, i_n=1}^2 \sum_{j_2, \dots, j_n=1}^2 \int_{\text{soft } P_s} \prod_{k=1}^n \left[\frac{d^4 P_k}{(2\pi)^4} (-)^{i_k+j_k} {}^*\Delta_{\xi_k \zeta_k}^{\text{mag}(i_k j_k)}(P_k) \right] \\ & \times \gamma^{\zeta_1} \diamond S^{(j_1 j_2)}(Q - K + P_1) \gamma^{\zeta_2} \diamond S^{(j_2 j_3)}(Q - K + \sum_{j=1}^2 P_j) \gamma^{\zeta_3} \dots \\ & \times \gamma^{\zeta_n} \diamond S^{(j_n \ell)}(Q - K + \sum_{j=1}^n P_j) \gamma^\mu \diamond S^{(\ell i_n)}(Q + \sum_{j=1}^n P_j) \gamma^{\xi_n} \dots \\ & \times \gamma^{\xi_3} \diamond S^{(i_3 i_2)}(Q + \sum_{j=1}^2 P_j) \gamma^{\xi_2} \diamond S^{(i_2 i_1)}(Q + P_1) \gamma^{\xi_1}. \end{aligned} \quad (4.3)$$

We substitute (3.9), (3.10), and (3.25) into (4.3). We then carry out the P_k -integration successively starting from P_1 -integration and then P_2 -integration and so on. From (4.3), pick out the term

$$\int dp_k p_k^2 \int dp_{k0} \frac{\rho_t(P_k)}{p_{k0}} \int d(\hat{\mathbf{p}}_k \cdot \hat{\mathbf{q}}) \diamond \tilde{S}_\tau^{(j_k j_{k+1})}(Q - K + \sum_{j=1}^k P_j) \diamond \tilde{S}_\tau^{(i_{k+1} i_k)}(Q + \sum_{j=1}^k P_j). \quad (4.4)$$

As has been noted above (and in Appendix A), the region, from which the leading contribution emerges, is

$$|p_{j0}| \ll p_j, \quad |\mathbf{p}_j \cdot \hat{\mathbf{q}}| \ll p_j \quad (1 \leq j \leq k-1).$$

Then we see that $|r_{k0} - \tau r_k| \ll p_k = O(gT)$, where $R_k \equiv Q + \sum_{j=1}^{k-1} P_j$. Thus for (4.4), we can use the result obtained in Appendix A:

$$\text{Eq. (4.4)} = \mathcal{S}_\tau^{j_k j_{k+1} i_{k+1} i_k}, \quad (i_{n+1} = j_{n+1} = \ell).$$

Using all this, we obtain

$$\begin{aligned} & \left(\Lambda_{gn}^{a;\mu}(Q - K, Q) \right)_{j_1 i_1}^\ell \\ &= -(-)^{i_1 + j_1 + \ell} \left[-\frac{g^2}{\pi^2} \left(\frac{N}{2} - C_F \right) T \right]^n \\ & \quad \times T^a \hat{Q}_\tau^\mu \gamma^0 \mathcal{S}_\tau^{(n)j_1 \ell i_1}, \end{aligned} \quad (4.5)$$

where

$$\begin{aligned} \mathcal{S}_\tau^{(n)ijk\ell} &\equiv \sum_{i_1, j_1=1}^2 (-)^{i_1 + j_1} \mathcal{S}_\tau^{(n-1)ij_1 i_1 \ell} \mathcal{S}_\tau^{j_1 j k i_1}, \\ & \left(\mathcal{S}_\tau^{(0)ijk\ell} \equiv \mathcal{S}_\tau^{ijk\ell} \right). \end{aligned}$$

Using the formulas in Appendix A, we can show by induction that

$$\mathcal{S}_\tau^{(n)ijk\ell} = \left(\frac{i\tau\pi}{2} L \right)^n \sum_{\sigma=\pm} \frac{\sigma^{n-1} a_\tau^{(\sigma)ijk\ell}}{(K \cdot \hat{R}_\tau + 2i\sigma\tau\Gamma)^n},$$

where $a_\tau^{(\sigma)}$ s are as in (A.5) in Appendix A. As in the case of $n = 1$, in the region $|K \cdot \hat{Q}_\tau| = O(\Gamma_q)$, $\left(\Lambda_{gn}^{a;\mu}(Q - K, Q) \right)$ s in (4.5) is of $O(1)$.

Now we inspect Fig. 5, which is obtained from Fig. 1 by replacing the bare quark-gluon vertexes with their multi-loop “corrections”, Fig. 4. Each contribution $\tilde{\Sigma}_F^{j_1 j_2 i_2 i_1}$ is of the same order of magnitude as the contribution of Fig. 1. As in the case of $n = 1$ above, from the formulas in Appendix A, we can readily derive

$$\sum_{i=1}^2 (-)^i \mathcal{S}_\tau^{(n)j i i \ell} \simeq 0.$$

Then, upon summation over j_1, j_2, i_1 , and i_2 , cancellation occurs between the contributions of Fig. 5 and the whole contribution, $\sum_{j_1, j_2, i_2, i_1=1}^2 \tilde{\Sigma}_F^{j_1 j_2 i_2 i_1}$, turns out to be nonleading.

We now turn to analyze a “crossed-ladder” diagram, which is obtained from Fig. 4 by interchanging the vertexes with thermal indexes j_1, j_2, \dots, j_n . Since the leading part of a soft-gluon propagator, Eqs. (3.10), is independent of the thermal indexes, the analysis of the “crossed-ladder” diagram does not bring about any complexity as compared to the ladder diagram Fig. 4. Then, Fig. 5 with “crossed-ladder” diagram(s) leads to nonleading $\tilde{\Sigma}_F$.

4.2 Analysis of Figs. 6 and 7

There is yet another diagram, Fig. 6, that leads to $O(1)$ contribution to the quark-gluon vertex. In Fig. 6, Q is hard and P and K are soft.

Fig. 6 yields

$$\begin{aligned} \left(\Lambda_{\text{Fig. 6}}^{a; \mu}(Q - K, Q) \right)_{ji}^{\ell} &= -\frac{i}{2} (-)^{i+j} g^2 N T^a \int_{\text{soft } P} \frac{d^4 P}{(2\pi)^4} \gamma_{\sigma} {}^{\circ} S^{(ji)}(Q + P) \gamma_{\rho} \\ &\quad \times {}^* \Delta^{\rho \xi(ii')}(P) {}^* \Delta^{\zeta \sigma(j'j)}(K + P) \left({}^* \mathcal{V}_{\xi \zeta}^{\mu}(K, P) \right)_{i'j'}^{\ell}. \end{aligned} \quad (4.6)$$

Here

$$\begin{aligned} \left({}^* \mathcal{V}_{\xi \zeta}^{\mu}(K, P) \right)_{i'j'}^{\ell} &= \left(\mathcal{V}_{\xi \zeta}^{(0); \mu}(K, P) \right)_{i'j'}^{\ell} \\ &\quad + \left({}^* \tilde{\mathcal{V}}_{\xi \zeta}^{\mu}(K, P) \right)_{i'j'}^{\ell} \\ \left(\mathcal{V}_{\xi \zeta}^{(0); \mu}(K, P) \right)_{i'j'}^{\ell} &= (-)^{\ell-1} \delta_{i'}^{\ell} \delta_{j'}^{\ell} \left\{ \delta_{\xi}^{\mu}(K - P)_{\zeta} \right. \\ &\quad \left. + g_{\xi \zeta}(K + 2P)^{\mu} - \delta_{\zeta}^{\mu}(2K + P)_{\xi} \right\}, \end{aligned}$$

where ${}^* \tilde{\mathcal{V}}_{\xi \zeta}^{\mu}$ is the HTL-contribution.

Let us estimate the order of magnitude of (4.6) in the region (3.23), $||q_0| - q| \leq O\{g^2 T \ln(g^{-1})\}$. We ignore possible factors of $\ln(g^{-1})$ and keep only powers of g . From (3.29), we have

$${}^{\circ} \rho_{\tau}(Q + P) \simeq \frac{1}{\pi} \frac{\gamma_q}{[q_0 - \tau q + p_0 - \tau \mathbf{p} \cdot \hat{\mathbf{q}}]^2 + \gamma_q^2},$$

where $\gamma_q \simeq \gamma_q(Q) = O(g^2 T)$. Then, when $|p_0 - \tau \mathbf{p} \cdot \hat{\mathbf{q}}| = O(g^2 T)$, ${}^{\circ} \rho_{\tau}(Q + P)$ is of $O\{1/(g^2 T)\}$. This is also the case for ${}^{\circ} S^{(ji)}(Q + P)$. Thus, $\int d^4 P = \int dp_0 \int dp p^2 \int d(\hat{\mathbf{p}} \cdot$

$\hat{\mathbf{q}}) = O(g^5 T^4)$. $^*\Delta$ s are of $O\{1/(g^3 T^2)\}$ and $^*\mathcal{V}_{\xi\zeta}^\mu$ is of $O(gT)$. Collecting all of them, we have

$$\left(\Lambda_{\text{Fig. 6}}^{a;\mu}(Q-K, Q)\right)_{ji}^\ell = O(1),$$

which is of the same order of magnitude as the bare quark-gluon vertex.

Let us turn to inspect Fig. 7, which is obtained from Fig. 1(a) by replacing the left bare quark-gluon vertex with Fig. 6. The same “phenomenon” as in the case of Fig. 3 occurs here. Each contribution of Fig. 7, which corresponds to a set of values of i, j, i' , and j' , is of the same order of magnitude as the contribution of Fig. 1. Noting again that all the soft-gluon propagator are independent of the thermal indexes and recalling the identity $\sum_{i,i',j'=1}^2 \left(^*\mathcal{V}_{\xi\zeta}^\mu(K, P)\right)_{ii'j'}^1 = 0$, we see that, upon summation over i, i' , and j' in Fig. 7, cancellation takes place between the contributions. Thus the contribution of Fig. 7 is nonleading.

5 Hard-gluon mode

The analysis goes parallel to that of Sec. III, so that we briefly present. The region of our interest is (3.1) or more precisely (3.23).

In Appendix B, computation of the one-loop contribution to $Re \tilde{\Pi}_F^{\nu\mu}(Q)$ and $Re \tilde{\Pi}_F^g(Q)$ is carried out in conventional hot QCD. The resultant $Re \tilde{\Pi}_F$ s are

$$Re \tilde{\Pi}_F^T(Q) \Big|_{\text{one-loops}} \simeq \frac{3}{2} m_T^2, \quad (5.1)$$

$$Re \tilde{\Pi}_F^L(Q) \Big|_{\text{one-loops}} \simeq Re \tilde{\Pi}_F^C(Q) \Big|_{\text{one-loops}} \simeq 0, \quad (5.2)$$

$$Re \tilde{\Pi}_F^D(Q) \Big|_{\text{one-loops}} = 0, \quad (5.3)$$

$$Re \tilde{\Pi}_F^g(Q) \Big|_{\text{one-loops}} \simeq 0, \quad (5.4)$$

where m_T is as in (3.17). The above results are gauge independent. Eqs. (5.2) and (5.4) mean that $|Re \Pi_F^A(Q)| \ll (gT)^2$ ($A = L, C, g$).

As in the case of hard-quark self-energy part, to leading order, the theory defined by (2.1) - (2.4) and (2.12) yields the same results, (5.1) - (5.4) (cf. Appendix B).

The self-consistency conditions $\Pi_F^{\mu\nu}(Q) = \tilde{\Pi}_F^{\mu\nu}(Q)$ and $\Pi_F^g(Q) = \tilde{\Pi}_F^g(Q)$ with $\Pi_F^{\mu\nu}(Q)$ in (2.3) and $\Pi_F^g(Q)$ in (2.4) yield

$$Re \Pi_F^T(Q) \simeq \frac{3}{2} m_T^2, \quad (5.5)$$

$$\begin{aligned} Re \Pi_F^L(Q) &\simeq Re \Pi_F^C(Q) \simeq Re \Pi_F^g(Q) \\ &\simeq 0. \end{aligned} \quad (5.6)$$

$$Re \Pi_F^D(Q) = 0, \quad (5.7)$$

which meet the requirements of BRS invariance, (2.11), (2.31), and (2.32).

Let us turn to analyze $Im \tilde{\Pi}_F(Q)$. It has been proved [10] that, in conventional hot QCD, the pole positions of the transverse and longitudinal propagators (cf. (2.21) and (2.22)) are independent of the choice of gauge. The diagram that yields the leading contribution to $Im \tilde{\Pi}_F^{\mu\nu}(Q)$ is depicted in Fig. 8(a), where K is soft.

Let us first compute the contribution from Fig. 8 to $Im \tilde{\Pi}_F^T(Q)$ and $Im \tilde{\Pi}_F^L(Q)$ in our theory. The contribution from Fig. 8(b) is

$$Im \tilde{\Pi}_F^{A(8b)}(Q) = -Im \Pi_F^A(Q) \quad (A = T, L).$$

For calculating Fig. 8(a), as in [5], for calculational ease, we use Coulomb gauge, in which only the transverse part of the hard-gluon propagator $\diamond\Delta^{\rho\sigma}(Q-K)$ contributes.

The leading contribution to $Im \tilde{\Pi}_F^T(Q)$ comes from Fig. 8(a) with magnetic part of the soft-gluon propagator. The same remark above after (3.10) applies here. Straight-forward calculation yields

$$\begin{aligned} Im \tilde{\Pi}_F^{T(8a)}(Q) &\simeq 2g^2 N q T \int_{\text{soft } K} \frac{d^4 K}{(2\pi)^3} \frac{\rho_t(K)}{k_0} \{1 - (\hat{\mathbf{q}} \cdot \hat{\mathbf{k}})^2\} \\ &\times \frac{g_T(q_0, q)}{[q_0 - \tau(q + \mathcal{M}_T^2/q) - k_0 + \tau \mathbf{k} \cdot \hat{\mathbf{q}}]^2 + [g_T(q_0, q)]^2}, \end{aligned}$$

where $\tau = q_0/|q_0|$, \mathcal{M}_T is as in (3.16), and

$$g_T(q_0, q) = \frac{1}{2q} Im \Pi_F^T(q_0, q).$$

Proceeding as in Sec. III, we obtain

$$Im \tilde{\Pi}_F^{T(8a)} \simeq \frac{g^2}{2\pi} N q T \frac{g_T(q_0, q)}{|g_T(q_0, q)|} \ln \left(\frac{m_T}{\Gamma_T(Q)} \right), \quad (5.8)$$

where

$$\Gamma_T(Q) \equiv \max \left[|q_0 - \tau (q + \mathcal{M}_T^2/q)|, |g_T(q_0, q)| \right].$$

Eq. (5.8) is valid at logarithmic accuracy. In conventional hot QCD, we have (5.8) with $g_T/|g_T| = -1$ and $\Gamma_T = |q_0 - \tau q|$, which is gauge independent. Then (5.8) is also gauge independent.

The self-consistency condition yields, in place of (3.24) in Sec. III,

$$\begin{aligned} \text{Im } \Pi_F^{T(8a)}(q_0, q) &= 2q g_T(q_0, q) \equiv -2q \gamma_T, \\ \gamma_T &= \frac{g^2}{4\pi} N T \ln(g^{-1}) \\ &\times \left[1 - \frac{\ln\{\ln(g^{-1})\}}{\ln(g^{-1})} + \mathcal{F} \right] + O(g^2 T). \end{aligned}$$

The contribution to $\text{Im } \tilde{\Pi}_F^{L(8a)}$ from Fig. 8(a) reads

$$\begin{aligned} \text{Im } \tilde{\Pi}_F^{L(8a)} &= -2g^2 N \frac{T}{q} \int_{\text{soft } K} \frac{d^4 K}{(2\pi)^3} \frac{g_T(q_0, q)}{[q_0 - \tau(q + \mathcal{M}_T^2/q) - k_0 + \tau \mathbf{k} \cdot \hat{\mathbf{q}}]^2 + [g_T(q_0, q)]^2} \\ &\times \left[\frac{\rho_t(K)}{k_0} \left\{ 1 + [\hat{\mathbf{k}} \cdot (\mathbf{q} - \mathbf{k})]^2 \right\} \right. \\ &\times \left\{ k_0^2 - (\hat{\mathbf{q}} \cdot \mathbf{k})^2 + \frac{2(Q - K)^2 - 2K^2 - Q^2}{4} - \frac{\{(Q - K)^2 - K^2\}^2}{4Q^2} \right\} \\ &+ \frac{\rho_\ell(K)}{k_0} \left\{ \left(q_0 - \frac{k_0}{2} \right)^2 \left[1 - \{\hat{\mathbf{q}} \cdot (\mathbf{q} - \mathbf{k})\}^2 \right] \right. \\ &\left. \left. - \left(1 - \frac{k_0^2}{4Q^2} \right) \left[k^2 - \{\mathbf{k} \cdot (\mathbf{q} - \mathbf{k})\}^2 \right] \right\} \right], \end{aligned}$$

where $\rho_\ell(K)$ is the spectral function of the electric part of the soft-gluon propagator.

Simple dimensional analysis yields

$$\begin{aligned} \text{Im } \tilde{\Pi}_F^{L(8a)} &= \text{Im } \Pi_F^L(Q) \\ &= O(g^4 T^2) + Q^2 \times O(g^2) + \frac{1}{Q^2} \times O(g^6 T^4), \end{aligned} \quad (5.9)$$

where factors of $\ln(g^{-1})$ are ignored. The first equality is due to the self-consistency condition.

The contribution from Fig. 8(a) to $\text{Im } \tilde{\Pi}_F^L$, where both K and $R - K$ are hard, may be analyzed similarly. The order of magnitude of the resultant contribution is again given by (5.9).

Eq. (5.9) shows that, at $|Q^2| = O(g^3 T^2)$, $|Im \Pi_F^L(Q)| = O(g^3 T^2)$, which is of the same order of magnitude as $|Q^2|$. In the region,

$$|Q^2| \gg g^3 T^2, \quad (5.10)$$

$|Im \Pi_F^L| \ll |Q^2|$, which means that the resummation of $Im \Pi_F^L$ is not necessary. Thus, in the region (5.10), we have

$$Im \Pi_F^L(Q) \simeq 0.$$

This together with (5.6) yields

$$\Pi_F^L(Q) \simeq 0, \quad (5.11)$$

$$\diamond \Delta_F^L(Q) \simeq \frac{1}{Q^2 + i0^+}. \quad (5.12)$$

Since this form is valid in the region (5.10), $+i0^+$ in the denominator is not necessary. Nevertheless we have kept it for the reason to be discussed below. As mentioned above, pole position of $\diamond \Delta_F^L(Q)$ is independent of the choice of gauge. Then the form (5.12) is gauge independent.

Eqs. (2.31) and (2.32) with (5.11) and (5.6) yield

$$\Pi_F^C(Q) \simeq \Pi_F^g(Q) \simeq 0, \quad (5.13)$$

which together with (2.23) and (2.19) leads to

$$\diamond \Delta_F^C(Q) \simeq 0, \quad (5.14)$$

$$\diamond \Delta_F^g(Q) \simeq \frac{1}{Q^2 + i0^+}. \quad (5.15)$$

The forms (5.13) - (5.15) are valid in the region (5.10).

Summarizing the result obtained above, we have for the diagonalized gluon- and FP-ghost-propagators,

$$\begin{aligned} \diamond \Delta_F^T(Q) &= \frac{1}{Q^2 - 3m_T^2/2 - iIm \Pi_F^T(Q)} \\ &\simeq \frac{\epsilon(q_0)}{2q} \frac{1}{q_0 - \epsilon(q_0)(q + \mathcal{M}_T^2/q) + i\epsilon(q_0) \gamma_T}, \end{aligned} \quad (5.16)$$

$$\begin{aligned} \diamond \Delta_F^L(Q) &\simeq \diamond \Delta_F^g(Q) \\ &\simeq \frac{1}{Q^2 + i0^+}, \end{aligned} \quad (5.17)$$

$$\diamond \Delta_F^C(Q) \simeq 0, \quad (5.18)$$

$$\diamond \Delta_F^D(Q) = \eta \frac{1}{Q^2 + i0^+}. \quad (5.19)$$

The 2×2 gluon propagator is related to (5.16) - (5.19) through the relations (2.25) - (2.27).

Here the similar observation as that at the end of Sec. III applies. The form of $Im \Delta_F^T(Q)$ is valid in the region (3.23). The forms of $Re \Delta_F^T(Q)$, $\Delta_F^L(Q)$, $\Delta_F^g(Q)$, and $\Delta_F^C(Q)$ are valid in the region $O(g^3T) < |q_0 - \epsilon(q_0)(q + \mathcal{M}_T^2/q)| \leq O\{g^2T \ln(g^{-1})\}$. For evaluating $Re \Delta_F^T(Q)$, $\Delta_F^L(Q)$, $\Delta_F^g(Q)$, and $\Delta_F^C(Q)$ in the region $|q_0 - \epsilon(q_0)(q + \mathcal{M}_T^2/q)| \leq O(g^3T)$, concrete evaluation of one-loop diagram as well as of two-loop diagram is necessary.

Suppose that we calculate some thermal amplitude. We encounter the integral,

$$\sum_{A=T, L, C, D, g} \int d^4Q \Delta_F^{A(rs)}(Q) \mathcal{H}^A(Q) \quad (r, s = 1, 2).$$

If this integral is insensitive to the region $|Q^2| \leq O(g^3T^2)$, we can use (5.17) for $\Delta_F^L(Q)$ and $\Delta_F^g(Q)$ and (5.18) for $\Delta_F^C(Q)$. This is because the phase-space volume of the region $|Q^2| \leq O(g^3T^2)$ is $O\{g/\ln(g^{-1})\}$ smaller than the volume of the region $O(g^3T^2) \leq |Q^2| \leq O\{g^2T^2 \ln(g^{-1})\}$. In the opposite case, we cannot use (5.17) and (5.18) and, as stated above, the analysis including the next-to-leading order is necessary.

Acknowledgment

This work was supported in part by the Grant-in-Aide for Scientific Research ((A)(1) (No. 08304024)) of the Ministry of Education, Science and Culture of Japan.

Appendix A

In this Appendix, we compute \mathcal{S}_τ^{ijkl} .

$$\begin{aligned} \mathcal{S}_\tau^{ijkl} &\equiv \int dp p^2 \int \frac{dp_0}{p_0} \rho_t(P) \\ &\times \int dz \circ \tilde{S}_\tau^{(ij)}(R - K + P) \circ \tilde{S}_\tau^{(kl)}(R + P), \end{aligned} \quad (A.1)$$

where, $z \equiv \tau \hat{\mathbf{p}} \cdot \hat{\mathbf{r}}$. In (A.1), R is hard and P and K are soft. We are interested in the form of \mathcal{S}_τ s in the region where $|r_0 - \tau r| = O(\Gamma_q) = O\{g^2T \ln(g^{-1})\}$ ($r_0 = \tau|r_0|$) and $|K \cdot \hat{R}_\tau| << gT$.

\mathcal{S}_τ^{1111} reads

$$\begin{aligned}
\mathcal{S}_\tau^{1111} &= \int dp p^2 \int \frac{dp_0}{p_0} \rho_t(P) \int_{-1}^1 dz \, {}^\diamond \tilde{S}_\tau^{(11)}(R - K + P) {}^\diamond \tilde{S}_\tau^{(11)}(R + P) \\
&\simeq \frac{1}{4} \int dp p^2 \int \frac{dp_0}{p_0} \rho_t(P) \int_{-1}^1 dz \left[\frac{1 - n_F}{r_0 - \tau r + p_0 - pz - K \cdot \hat{R}_\tau + i\tau\Gamma_q} \right. \\
&\quad \left. + \frac{n_F}{r_0 - \tau r + p_0 - pz - K \cdot \hat{R}_\tau - i\tau\Gamma_q} \right] \\
&\quad \times \left[\frac{1 - n_F}{r_0 - \tau r + p_0 - pz + i\tau\Gamma_q} + \frac{n_F}{r_0 - \tau r + p_0 - pz - i\tau\Gamma_q} \right] \\
&\simeq \frac{1}{4} \int dp p \int \frac{dp_0}{p_0} \rho_t(P) \\
&\quad \times \sum_{\sigma=\pm} \left[\frac{n_F(1 - n_F)}{K \cdot \hat{R}_\tau + 2i\sigma\tau\Gamma_q} \left\{ \ln \left(\frac{-p + p_0 + r_0 - \tau r + i\sigma\tau\Gamma_q}{p + p_0 + r_0 - \tau r + i\sigma\tau\Gamma_q} \right) \right. \right. \\
&\quad \left. \left. + \ln \left(\frac{p + p_0 + r_0 - \tau r - K \cdot \hat{R}_\tau - i\sigma\tau\Gamma_q}{-p + p_0 + r_0 - \tau r - K \cdot \hat{R}_\tau - i\sigma\tau\Gamma_q} \right) \right\} \right. \\
&\quad \left. + \frac{\{\theta(-\sigma) - n_F\}^2}{K \cdot \hat{R}_\tau} \left\{ \ln \left(\frac{-p + p_0 + r_0 - \tau r - i\sigma\tau\Gamma_q}{p + p_0 + r_0 - \tau r - i\sigma\tau\Gamma_q} \right) \right. \right. \\
&\quad \left. \left. + \ln \left(\frac{p + p_0 + r_0 - \tau r - K \cdot \hat{R}_\tau - i\sigma\tau\Gamma_q}{-p + p_0 + r_0 - \tau r - K \cdot \hat{R}_\tau - i\sigma\tau\Gamma_q} \right) \right\} \right] ,
\end{aligned}$$

where $n_F \equiv n_F(q)$. The quantity in the second curly brackets leads to nonleading contribution. As in the case of self-energy part in Sec. III, at logarithmic accuracy, the region $|p_0| \ll p$ yields the dominant contribution. Then we drop the factors p_0 in the arguments of logarithms. Using (3.16) and integrating over $-\beta p < p_0 < +\beta p$ with $\beta \ll 1$, we obtain

$$\begin{aligned}
\mathcal{S}_\tau^{1111} &= \frac{1}{2\pi} \int \frac{dp}{p} \arctan \left(\frac{\pi\beta\mathcal{M}_T^2}{p^2} \right) \\
&\quad \times \sum_{\sigma=\pm} \left[\frac{n_F(1 - n_F)}{K \cdot \hat{R}_\tau + 2i\sigma\tau\Gamma_q} \left\{ \ln \left(\frac{-p + p_0 + r_0 - \tau r + i\sigma\tau\Gamma_q}{p + p_0 + r_0 - \tau r + i\sigma\tau\Gamma_q} \right) \right. \right. \\
&\quad \left. \left. + \ln \left(\frac{p + p_0 + r_0 - \tau r - K \cdot \hat{R}_\tau - i\sigma\tau\Gamma_q}{-p + p_0 + r_0 - \tau r - K \cdot \hat{R}_\tau - i\sigma\tau\Gamma_q} \right) \right\} \right] .
\end{aligned}$$

Let us inspect the first logarithmic function with $p_0 = 0$, $\ln(\dots)$. For $p = O(gT)$, $\ln(\dots) \simeq i\sigma\tau\pi$, $\ln(\dots) = 0$ at $p = 0$, and the transition region is $p = O(\Gamma_q)$. Similar observation may be made for the second logarithmic function.

Then, we obtain, at logarithmic accuracy,

$$\mathcal{S}_\tau^{1111} \simeq \frac{i\tau\pi}{2} L \sum_{\sigma=\pm} \frac{\sigma n_F(1-n_F)}{K \cdot \hat{R}_\tau + 2i\sigma\tau\Gamma_q}, \quad (\text{A.2})$$

where

$$L \equiv \frac{1}{2} \left[\ln \left(\frac{m_T}{\Gamma_q} \right) + \ln \left(\frac{m_T}{\max(\Gamma_q, |K \cdot \hat{R}_\tau|)} \right) \right]. \quad (\text{A.3})$$

It is to be noted that, at logarithmic accuracy, the restriction of the z region to $|z| \ll 1$ or $|\hat{\mathbf{p}} \cdot \hat{\mathbf{r}}| \ll p$ yields the same result (A.2).

In a similar manner, we can compute all \mathcal{S}_τ^{ijkl} . Writing

$$\mathcal{S}_\tau^{ijkl} \simeq \frac{i\tau\pi}{2} L \sum_{\sigma=\pm} \frac{a_\tau^{(\sigma)ijkl}}{K \cdot \hat{R}_\tau + 2i\sigma\tau\Gamma_q}, \quad (\text{A.4})$$

we have

$$\begin{aligned} a_\tau^{(\pm)1111} &= a_\tau^{(\pm)2222} = a_\tau^{(\pm)1221} = a_\tau^{(\pm)2112} \\ &= \pm n_F[1 - n_F], \\ a_\tau^{(\pm)1112} &= a_{-\tau}^{(\pm)1121} = -a_{-\tau}^{(\mp)2221} = -a_\tau^{(\mp)2212} \\ &= -a_\tau^{(\mp)1211} = -a_{-\tau}^{(\mp)2111} = a_{-\tau}^{(\pm)2122} = a_\tau^{(\pm)1222} \\ &= \mp[\theta(-\tau) - n_F][\theta(\mp) - n_F], \\ a_\tau^{(\pm)1122} &= -a_\tau^{(\mp)2211} = \mp[\theta(\mp) - n_F]^2 \\ a_\tau^{(\pm)1212} &= a_{-\tau}^{(\pm)2121} \\ &= \mp[\theta(-\tau) - n_F]^2. \end{aligned} \quad (\text{A.5})$$

Appendix B Computation of the real part of $\tilde{\Pi}_F^{\nu\mu}(Q)$

In this Appendix, we compute the real part of the one-loop thermal gluon self-energy part, $Re \tilde{\Pi}_F^{(g)\nu\mu}(Q)$, in conventional hot QCD. The region of our interest is

$$|Q^\mu| = O(T), \quad |Q^2| \leq O\{g^2 T^2 \ln(g^{-1})\}. \quad (\text{B.1})$$

1. Contribution of a quark loop

The contribution of a quark loop reads

$$Re \tilde{\Pi}_F^{(q)\nu\mu}(Q) = -\frac{g^2}{2} N_f Re \int \frac{d^4 K}{(2\pi)^4} tr \left[\gamma^\mu S^{(11)}(K-Q) \gamma^\nu S^{(11)}(K) \right]. \quad (B.2)$$

After straightforward manipulations, we have for $Re \tilde{\Pi}_F^{(q)T}(Q)$,

$$\begin{aligned} Re \tilde{\Pi}_F^{(q)T}(Q) &= \frac{g^2}{12} N_f T^2 \\ &\quad - \frac{g^2}{4\pi^2} N_f \frac{Q^2}{q^3} \int_0^\infty dk k^2 n_F(k) \sum_{\tau=\pm} \ln \left(\frac{|Q^2 + 2qk - 2\tau q_0 k|}{|Q^2 - 2qk - 2\tau q_0 k|} \right) \\ &\quad + \frac{g^2}{8\pi^2} N_f \frac{Q^2}{q^2} \int_0^\infty dk n_F(k) \\ &\quad \times \left[4k - \sum_{\tau=\pm} \frac{2q^2 + Q^2 - 4\tau q_0 k}{2q} \ln \left(\frac{|Q^2 + 2qk - 2\tau q_0 k|}{|Q^2 - 2qk - 2\tau q_0 k|} \right) \right]. \end{aligned} \quad (B.3)$$

It can easily be shown that, in the region (B.1), no $O(g^2 T^2)$ contribution emerges from the second and third terms on the R.H.S. Then we have

$$Re \tilde{\Pi}_F^{(q)T}(Q) \simeq \frac{g^2}{12} N_f T^2. \quad (B.4)$$

We next have

$$\begin{aligned} Re \left(\tilde{\Pi}_F^{(q)L}(Q) + 2\tilde{\Pi}_F^{(q)T}(Q) \right) &= Re \Pi_F^{(q)\nu\mu}(Q) \left(g_{\mu\nu} - \frac{Q_\mu Q_\nu}{Q^2} \right) \\ &= \frac{g^2}{6} N_f T^2 \\ &\quad - \frac{g^2}{4\pi^2} N_f \frac{Q^2}{q} \int_0^\infty dk n_F(k) \\ &\quad \times \sum_{\tau=\pm} \ln \left(\frac{|Q^2 + 2qk - 2\tau q_0 k|}{|Q^2 - 2qk - 2\tau q_0 k|} \right) \\ &\simeq \frac{g^2}{6} N_f T^2. \end{aligned} \quad (B.5)$$

From (B.4) and (B.5), we obtain

$$Re \tilde{\Pi}_F^{(q)L}(Q) \simeq 0. \quad (B.6)$$

$Re \tilde{\Pi}_F^{(q)\nu\mu}(Q)$ in (B.2) satisfies $Q_\nu \tilde{\Pi}_F^{(q)\nu\mu}(Q) = 0$, so that

$$Re \tilde{\Pi}_F^{(q)C}(Q) = Re \tilde{\Pi}_F^{(q)D}(Q) = 0.$$

It is to be noted that the integrals in (B.3) and (B.5) are insensitive to the region $K \simeq 0$ or $Q - K \simeq 0$. Then the replacement $S^{(11)}(K) \rightarrow {}^*S^{(11)}(K)$ [$S^{(11)}(Q - K) \rightarrow {}^*S^{(11)}(Q - K)$] in the soft- K [soft-($Q - K$)] region in (B.2) does not bring about the contribution of $O(g^2 T^2)$.

Also to be noted is that the leading contributions, (B.4) and (B.6), have come from the hard- K and hard- $(Q - K)$ region. From the above derivation, we can easily verify that the formula (B.2) with ${}^\circ S^{(11)}$ s for $S^{(11)}$ s leads to the same leading-order results (B.4) and (B.6).

2. Contribution of gluon loops and a FP-ghost loop

The (1, 1)-component of the bare thermal gluon propagator is written as

$$\begin{aligned} \Delta_{\mu\nu}^{(11)}(Q) &= - \left[g_{\mu\nu} - (\eta - 1) Q_\mu Q_\nu \frac{\partial}{\partial \lambda^2} \right] \\ &\quad \times \Delta^{(11)}(Q; \lambda^2) \Big|_{\lambda=0}, \quad (j, \ell = 1, 2), \\ \Delta^{(11)}(Q; \lambda^2) &= \frac{1}{Q^2 - \lambda^2 + i0^+} \\ &\quad - 2\pi i n_B(|q_0|) \delta(Q^2 - \lambda^2), \end{aligned}$$

where $n_B(x) \equiv 1/(e^{x/T} - 1)$. Accordingly the contribution to $\tilde{\Pi}_F^{\nu\mu}$ of gluon loops plus the contribution of FP-ghost loop consists of three terms,

$$\begin{aligned} \tilde{\Pi}_F^{\nu\mu}(Q) &= \tilde{\Pi}_F^{\nu\mu(0)}(Q) + (\eta - 1) \tilde{\Pi}_F^{\nu\mu(1)}(Q) \\ &\quad + (\eta - 1)^2 \tilde{\Pi}_F^{\nu\mu(2)}(Q), \end{aligned}$$

and the one-loop contribution to the FP-ghost self-energy part $\tilde{\Pi}_F^g(Q)$ consists of two terms

$$\tilde{\Pi}_F^g(Q) = \tilde{\Pi}_F^{g(0)}(Q) + (\eta - 1) \tilde{\Pi}_F^{g(1)}(Q). \quad (\text{B.7})$$

We summarize the result of the straightforward calculation. $\tilde{\Pi}_F^{(j)\nu\mu}(Q)$ ($j = 0, 1, 2$) satisfies the relation,

$$Q_\nu \tilde{\Pi}_F^{\nu\mu(0)}(Q) = Q_\nu \tilde{\Pi}_F^{\nu\mu(2)}(Q) = Q_\nu Q_\mu \tilde{\Pi}_F^{\nu\mu(1)}(Q) = 0,$$

so that, with obvious notations,

$$\begin{aligned} Re \tilde{\Pi}_F^{C(0)} &= Re \tilde{\Pi}_F^{D(0)} = Re \tilde{\Pi}_F^{D(1)} = Re \tilde{\Pi}_F^{C(2)} \\ &= Re \tilde{\Pi}_F^{D(2)} = 0. \end{aligned}$$

Nonvanishing $Re \tilde{\Pi}_F$ s are

$$\begin{aligned} &Re \tilde{\Pi}_F^{T(0)}(Q) \\ &= \frac{g^2}{6} N T^2 - \frac{g^2}{4\pi^2} N \frac{Q^2}{q^3} \int_0^\infty dk k^2 n_B(k) \sum_{\tau=\pm} \ln \left(\frac{|Q^2 + 2qk - 2\tau q_0 k|}{|Q^2 - 2qk - 2\tau q_0 k|} \right) \\ &\quad + \frac{g^2}{8\pi^2} N \frac{Q^2}{q^2} \int_0^\infty dk n_B(k) \\ &\quad \times \left[4k - \sum_{\tau=\pm} \frac{4q^2 + Q^2 - 4\tau q_0 k}{2q} \ln \left(\frac{|Q^2 + 2qk - 2\tau q_0 k|}{|Q^2 - 2qk - 2\tau q_0 k|} \right) \right], \end{aligned} \quad (B.8)$$

$$\begin{aligned} &Re \left(\tilde{\Pi}_F^{L(0)}(Q) + 2\tilde{\Pi}_F^{T(0)}(Q) \right) \\ &= \frac{g^2}{3} N T^2 - \frac{5}{8\pi^2} g^2 N \frac{Q^2}{q} \int_0^\infty dk n_B(k) \sum_{\tau=\pm} \ln \left(\frac{|Q^2 + 2qk - 2\tau q_0 k|}{|Q^2 - 2qk - 2\tau q_0 k|} \right), \end{aligned} \quad (B.9)$$

$$\begin{aligned} &Re \tilde{\Pi}_F^{T(1)}(Q) \\ &= \frac{g^2}{16\pi^2} N \frac{Q^2}{q} \frac{\partial}{\partial \mu^2} \int_0^\infty dk \frac{k}{\sqrt{k^2 + \mu^2}} n_B(\sqrt{k^2 + \mu^2}) \\ &\quad \times \sum_{\tau=\pm} \left[\left\{ Q^2 \left(1 + \frac{k^2}{q^2} \right) + \frac{(q^2 + \mu^2)^2 - 4\tau q_0 \sqrt{k^2 + \mu^2} (Q^2 + \mu^2) + 4q_0^2 \mu^2}{4q^2} \right\} \right. \\ &\quad \times \left. L_\tau(\mu^2, \lambda^2 = 0) - 4qk - \frac{k}{q} (Q^2 + \mu^2) \right] \Big|_{\mu^2=0} \\ &\quad + \frac{g^2}{16\pi^2} N \frac{Q^2}{q} \frac{\partial}{\partial \lambda^2} \int_0^\infty dk n_B(k) \\ &\quad \times \sum_{\tau=\pm} \left[\left\{ Q^2 \left(1 + \frac{k^2}{q^2} \right) + \frac{(Q^2 - \lambda^2)(Q^2 - \lambda^2 - 4\tau q_0 k)}{4q^2} \right\} \right. \\ &\quad \times \left. L_\tau(\mu^2 = 0, \lambda^2) - \frac{k}{q} (Q^2 - \lambda^2) \right] \Big|_{\lambda^2=0}, \end{aligned} \quad (B.10)$$

$$\begin{aligned} &Re \left(\tilde{\Pi}_F^{L(1)}(Q) + 2\tilde{\Pi}_F^{T(1)}(Q) \right) \\ &= \frac{5g^2}{32\pi^2} N \frac{Q^2}{q} \frac{\partial}{\partial \mu^2} \int_0^\infty dk \frac{k}{\sqrt{k^2 + \mu^2}} n_B(\sqrt{k^2 + \mu^2}) \\ &\quad \times \sum_{\tau=\pm} \left[\left(Q^2 + \frac{2}{5} \mu^2 \right) L_\tau(\mu^2, \lambda^2 = 0) - 4qk \right] \Big|_{\mu^2=0} \end{aligned}$$

$$+ \frac{5g^2}{32\pi^2} N \frac{Q^2}{q} \frac{\partial}{\partial \lambda^2} \int_0^\infty dk n_B(k) \left(Q^2 + \frac{2}{5} \lambda^2 \right) \sum_{\tau=\pm} L_\tau(\mu^2 = 0, \lambda^2) \Big|_{\lambda^2=0}, \quad (\text{B.11})$$

$$\begin{aligned} & Re \tilde{\Pi}_F^{C(1)}(Q) \\ &= -\frac{\sqrt{2}g^2}{32\pi^2} N \frac{Q^2}{q^2} \frac{\partial}{\partial \mu^2} \int_0^\infty dk \frac{k}{\sqrt{k^2 + \mu^2}} n_B(\sqrt{k^2 + \mu^2}) \\ &\quad \times \left[4kq q_0 + Q^2 \sum_{\tau=\pm} \left(\tau \sqrt{k^2 + \mu^2} - \frac{q_0}{2} \right) L_\tau(\mu^2, \lambda^2 = 0) \right] \Big|_{\mu^2=0} \\ &\quad + \frac{\sqrt{2}g^2}{32\pi^2} N \frac{Q^4}{q^2} \frac{\partial}{\partial \lambda^2} \int_0^\infty dk n_B(k) \sum_{\tau=\pm} \left(\tau k - \frac{q_0}{2} \right) L_\tau(\mu^2 = 0, \lambda^2) \Big|_{\lambda^2=0}, \\ &\quad + \frac{\sqrt{2}g^2}{32\pi^2} N Q^2 \frac{q_0}{q^2} \int_0^\infty dk n_B(k) L_\tau(\mu^2 = 0, \lambda^2 = 0), \end{aligned} \quad (\text{B.12})$$

$$\begin{aligned} & Re \tilde{\Pi}_F^{T(2)}(Q) \\ &= \frac{g^2}{32\pi^2} N \frac{Q^4}{q^3} \frac{\partial}{\partial \mu^2} \frac{\partial}{\partial \lambda^2} \int_0^\infty dk \frac{k}{\sqrt{k^2 + \mu^2}} n_B(\sqrt{k^2 + \mu^2}) \\ &\quad \times \sum_{\tau=\pm} \left[(Q^2 + \mu^2 - \lambda^2) qk \right. \\ &\quad \left. - \frac{4k^2 Q^2 + (Q^2 + \mu^2 - \lambda^2)^2 - 4\tau q_0 \sqrt{k^2 + \mu^2} (Q^2 + \mu^2 - \lambda^2) + 4q_0^2 \mu^2}{4} \right. \\ &\quad \left. \times L_\tau(\mu^2, \lambda^2) \right] \Big|_{\mu^2 = \lambda^2 = 0}, \end{aligned} \quad (\text{B.13})$$

$$\begin{aligned} & Re \left(\tilde{\Pi}_F^{L(2)}(Q) + 2\tilde{\Pi}_F^{T(2)}(Q) \right) \\ &= \frac{g^2}{64\pi^2} N \frac{Q^4}{q} \frac{\partial}{\partial \mu^2} \frac{\partial}{\partial \lambda^2} \int_0^\infty dk \frac{k}{\sqrt{k^2 + \mu^2}} n_B(\sqrt{k^2 + \mu^2}) \\ &\quad \times \sum_{\tau=\pm} \left[-4qk \frac{Q^2 + \mu^2 - \lambda^2}{Q^2} + \{Q^2 - 2(\mu^2 + \lambda^2)\} L_\tau(\mu^2, \lambda^2) \right] \Big|_{\mu^2 = \lambda^2 = 0}. \end{aligned} \quad (\text{B.14})$$

Here

$$L_\tau(\mu^2, \lambda^2) \equiv \ln \left(\frac{|Q^2 + 2qk - 2\tau q_0 \sqrt{k^2 + \mu^2} + \mu^2 - \lambda^2|}{|Q^2 - 2qk - 2\tau q_0 \sqrt{k^2 + \mu^2} + \mu^2 - \lambda^2|} \right).$$

$\tilde{\Pi}_F^{g(j)}(Q)$ ($j = 0, 1$) in (B.7) reads

$$Re \tilde{\Pi}_F^{g(0)}(Q) = -\frac{g^2}{16\pi^2} N \frac{Q^2}{q} \int_0^\infty dk n_B(k) \sum_{\tau=\pm} \ln \left(\frac{|Q^2 + 2qk - 2\tau q_0 k|}{|Q^2 - 2qk - 2\tau q_0 k|} \right) \quad (\text{B.15})$$

$$Re \tilde{\Pi}_F^{g(1)}(Q) = \frac{g^2}{64\pi^2} N \frac{Q^2}{q} \frac{\partial}{\partial \mu^2} \int_0^\infty dk \frac{k}{\sqrt{k^2 + \mu^2}} n_B(\sqrt{k^2 + \mu^2})$$

$$\begin{aligned}
& \times \sum_{\tau=\pm} \left[Q^2 L_\tau(\mu^2, \lambda^2 = 0) - 4qk \right] \Big|_{\mu^2=0} \\
& + \frac{g^2}{64\pi^2} N \frac{Q^4}{q} \frac{\partial}{\partial \lambda^2} \int_0^\infty dk n_B(k) \sum_{\tau=\pm} L_\tau(\mu^2 = 0, \lambda^2) \Big|_{\lambda^2=0}.
\end{aligned} \tag{B.16}$$

As in the previous subsection 1, we can easily see that no $O(g^2 T^2)$ contribution arises from the integrals in (B.8), (B.9), and (B.15). Computation of (B.10) - (B.14) and (B.16) goes as follows. Take derivative with respect to μ^2 and/or λ^2 and set $\mu^2 = \lambda^2 = 0$. Divide the integration region into $0 \leq k \leq k^*$ and $k^* \leq k$, where $gT < k^* < T$. It can readily be shown that the contributions from the latter region are nonleading when compared to (B.4) and (B.5). The contributions from the former region may be calculated explicitly by using $n_B(k) \simeq T/k$ and $n_F(k) \simeq 1/2$ and are shown to be also nonleading.

After all this, we have

$$Re \tilde{\Pi}_F^T(Q) \simeq \frac{g^2}{6} N T^2, \tag{B.17}$$

$$Re \tilde{\Pi}_F^L(Q) \simeq \tilde{\Pi}_F^C(Q) \simeq \tilde{\Pi}_F^g(Q) \simeq 0, \tag{B.18}$$

$$Re \tilde{\Pi}_F^D(Q) = 0, \tag{B.19}$$

which are gauge independent. Similar observation as that at the end of subsection 1 applies here.

Using (B.4), (B.6), and (B.17) - (B.19), we finally obtain

$$\begin{aligned}
Re \tilde{\Pi}_F^T(Q) & \simeq \frac{1}{6} \left(N + \frac{N_f}{2} \right) (gT)^2 \left(= \frac{3}{2} m_T^2 \right), \\
Re \tilde{\Pi}_F^L(Q) & \simeq Re \tilde{\Pi}_F^C(Q) \simeq Re \tilde{\Pi}_F^g(Q) \simeq 0. \\
Re \tilde{\Pi}_F^D(Q) & = 0.
\end{aligned}$$

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Figure captions

Fig. 1. Diagrams for the self-energy part of the quark mode.

Fig. 2. An one-loop diagram for the quark-gluon vertex. “ ℓ ”, “ i ”, and “ j ” are thermal indexes. The blob on the gluon line indicates the effective soft-gluon propagator.

Fig. 3. A two-loop diagram for the self-energy part of the quark mode.

Fig. 4. An n -loop diagram for the quark-gluon vertex. “ ℓ ”, “ i_1 ” – “ i_n ” and “ j_1 ” – “ j_n ” are thermal indexes.

Fig. 5. A multi-loop diagram for the self-energy part of the quark mode.

Fig. 6. An one-loop diagram for the quark-gluon vertex. The blob on the vertex indicates the effective soft tri-gluon vertex.

Fig. 7. A two-loop diagram for the self-energy part of the quark mode.

Fig. 8. Diagrams for the self-energy part of the gluon mode.

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